

E-5-X

Roll No.....

Total No. of Questions : 26]

[Total No. of Printed Pages : 7

XIIWZJDRO/N19**24905-X****MATHEMATICS**

Time : 3 Hours]

[Maximum Marks : 100

(Long Answer Type Questions)

6 each

1. Using properties of determinants prove that :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

Or

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} . Using A^{-1} solve the system of

equations :

$$2x - 3y + 5z = 11, 3x + 2y - 4z = -5, x + y - 2z = -3$$

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Turn Over

2. Find $\frac{dy}{dx}$ if $y = (\log x)^x + x^{\log x}$.

Or

If $y = (\tan^{-1} x)^2$ show that :

$$(x^2 + 1)^2 y_2 + 2x (x^2 + 1) y_1 = 2$$

3. Evaluate $\int_0^2 (x^2 + 1) dx$ as the limit of a sum.

Or

Evaluate :

$$\int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$$

4. Find the equation of the plane passing through three points (1, 1, 0), (1, 2, 1) and (-2, 2, -1).

Or

Find the shortest distance between the lines whose vector equations are :

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$$

and

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

5. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Or

Find the variance and S.D. of a number obtained on throwing an unbiased die.

(Short Answer Type Questions)

4 each

6. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $(f \circ f) = x$ for all $x \neq \frac{2}{3}$. What is f^{-1} ?

7. Prove that :

$$\cot^{-1} \left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4} \right)$$

8. If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, verify that $A \cdot \text{adj } A = |A| \cdot I$

9. Find $\frac{dy}{dx}$ when :

$$x = a(\cos \theta + \theta \sin \theta)$$

$$y = a(\sin \theta - \theta \cos \theta)$$

10. Find the equation of a tangent to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x - y + 9 = 0$.

11. Find the value of 'k' so that the function $f(x)$ defined by :

$$f(x) = \begin{cases} kx+1, & \text{if } x \leq 5 \\ 3x-5, & \text{if } x > 5 \end{cases}$$

is continuous at $x = 5$.

12. Find a vector of magnitude 5 units and parallel to the resultant of

vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

13. Evaluate :

$$\int_0^{\pi/2} \sin^3 x \, dx$$

14. Solve the homogeneous differential equation :

$$(x^2 - y^2) \, dx + 2xy \, dy = 0$$

15. Solve graphically to :

Maximize :

$$Z = 3x + 2y$$

Subject to constraints :

$$x + 2y \leq 10$$

$$3x + y \leq 15$$

$$x, y \geq 0$$

(Very Short Answer Type Questions)

2 each

16. For the matrix $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ verify $A'A = I$.

17. Find the rate of change of the area of a circle with respect to its radius r when $r = 3$ cm.

18. Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point whose x -coordinate is 2.

19. Find $\frac{dy}{dx}$ if $xy + y^2 = \tan x + y$.

20. Evaluate :

$$\int \frac{1 - \cos x}{1 + \cos x} dx$$

21. Find the general solution of the differential equation :

$$\frac{dy}{dx} = \sqrt{4 - y^2}, \quad |y| < 2$$

22. Find the unit vector in the direction of PQ, where P and Q are points (1, 2, 3) and (4, 5, 6) respectively.

23. Find the angle between the vectors \vec{a} and \vec{b} with magnitude 1 and 2 respectively and $\vec{a} \cdot \vec{b} = 1$.

24. Find $|\vec{a} \times \vec{b}|$ if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 3\hat{k}$.

25. Compute $P(A/B)$ if $P(B) = 0.5$ and $P(A \cap B) = 0.32$

(Objective Type Questions)

1 each

26. (i) Define a symmetric matrix.

(ii) What is linear programme problem ?

(iii) Define Feasible region of LPP.

(iv) Define a symmetric relation.

(v) A die is rolled once. The probability of obtaining even prime

number is (Fill in the blank)

(vi) The direction cosines of x-axis are

(Fill in the blank)

(vii) A coin is tossed thrice, then probability of exactly 2 heads is

.....

(Fill in the blank)

(viii) $\frac{d}{dx}(\log \sec x) = \sec x$.

(True/False)

(ix) $\cos\left(\frac{\pi}{6} - \sin^{-1}\frac{1}{2}\right)$ is :

(A) 0

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) 1

(x) $\int \tan x dx = \dots\dots\dots$

(Fill in the blank)