B–21–C

Roll No. ....

Total No. of Questions : 26] [Total No. of Printed Pages : 8 + Graph

HSEIIRWZJO17

15221–C

MATHEMATICS

Time : 3 Hours] **[Maximum Marks : 100**

(Long Answer Type Questions) 6 each

1. If x, y, z are different and

$$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0,$$

then show that $1 + xyz = 0$.

Or

If :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1}.

2. Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$.

Or

If $x = \sqrt{a^{\cos^{-1} t}}$ $y = \sqrt{a^{\sin^{-1} t}}$, show that :

$$\frac{dy}{dx} = -\frac{y}{x}$$

3. Evaluate :

$$\int_0^{\pi/2} \sqrt{\sin\phi}\,\cos^5\phi\, d\phi$$

Or

Evaluate :

$$\int \frac{(3\sin\phi - 2)\cos\phi}{5 - \cos^2\phi - 4\sin\phi}\, d\phi$$

4. Find the value of P so that the lines :

$$\frac{1-x}{3} = \frac{7y-14}{2P} = \frac{z-3}{2}$$

and $$\frac{7-7x}{3P} = \frac{y-5}{1} = \frac{6-z}{5}$$

are at right angles.

Or

Find the vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=6$ and $\vec{r}\cdot\left(2\hat{i}+3\hat{j}+4\hat{k}\right)=-5$ and the point (1, 1, 1).

5. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective ?

Or

Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of kings.

(Short Answer Type Questions) 4 each

6. Write $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$, $0 < x < \pi$ in the simplest form.

7. Consider $f : R \rightarrow R$ given by $f(x) = 4x + 3$. Show that f is invertible. Find the inverse of f.

8. Find x and y, if :

$$x + y = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix}$$

and
$$x - y = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix}$$

9. Show that :

$$y = \log (1 + x) - \frac{2x}{2+x}, \ x > -1,$$

is an increasing function of x throughout its domain.

10. Find the relation between a and b so that the function f is defined by

$$f(x) = \begin{cases} ax+1, & \text{if } \ x \leq 3 \\ bx+3, & \text{if } \ x > 3 \end{cases}$$

is continuous at $x = 3$.

11. The radius of a circle is increasing at the rate of 0·7 cm/s. What is the rate of increase of its circumference ?

12. Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.

13. Find the general solution of the differential equation :

$$x\frac{dy}{dx} + 2y = x^2 (x \neq 0)$$

14. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant.

15. Determine graphically the minimum value of the objective function

$$Z = -50x + 20y$$

Subject to the constraints :

$$2x - y \geq -5;$$

$$3x + y \geq 3;$$

$$2x - 3y \leq 12,$$

$$x \geq 0,\ y \geq 0$$

(Very Short Answer Type Questions) 2 each

16. Compute :

$$\begin{pmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{pmatrix}$$

17. Find $\frac{dy}{dx}$, if $ax + by^2 = \cos y$.

18. Use differential to approximate $\sqrt{49 \cdot 5}$.

19. Find the slope of tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

20. Find $\frac{dy}{dx}$, if $x = a \cos \theta$, $y = b \cos \theta$.

21. Find the general solution of the differential equation :

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

22. Find the direction cosines of the vector $\hat{i} + \hat{j} + 2\hat{k}$.

23. Find the projection of the vector $2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\hat{i} + 2\hat{j} + \hat{k}$.

24. Find $|\vec{a} \times \vec{b}|$, if

$$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$$

and

$$\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$$

25. Compute $P\left(\frac{F}{E}\right)$, if $P(E) = 0{\cdot}6$ and $P(E \cap F) = 0{\cdot}2$.

(Objective Type Questions) 1 each

26. (i) Define Identity matrix.

(ii) Define the term feasible solution.

(iii) What do you mean by transportation problems ?

(iv) Give an example of a relation which is symmetric and transitive but not reflexive..

(v) The direction ratios of the line joining the points (3, –2, 1) and (2, –4, 3) are ....

(vi) If A and B are disjoint events, then $P\left(\frac{A \cup B}{F}\right)$ = ...

(vii) In a single throw of two dice, the probability of an odd number on the first and 6 on the second is $\frac{1}{2}$. (True/False)

(viii) $\frac{d}{dx}[\log \cot x] = -\sec x \operatorname{cosec} x$. (True/False)

(ix) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to :

(a) $\frac{7\pi}{6}$ (b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

(Choose the correct option)

(x) $\int \text{cosec}\, x \, dx$ is equal to :

(a) $\log |\text{cosec}\, x + \cot x| + C$

(b) $\log \left|\tan \frac{x}{2}\right| + C$

(c) $\log \left|\cot \frac{x}{2}\right| + C$

(d) None of these (Choose the correct option)

F–5–Y

Roll No. ....

Total No. of Questions : 26]

[Total No. of Printed Pages : 8

12thRWZJO18

20105–Y

MATHEMATICS

Time : 3 Hours]

[Maximum Marks : 100

(Long Answer Type Questions)

6 each

1. Using properties of determinants prove that :

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$$

Or

Solve system of linear equations, using matrix method :

$$x - y + 2z = 7$$

$$3x + 4y - 5z = -5$$

$$2x - y + 3z = 12$$

2. Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$.

Or

If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2y_2 + xy_1 + y = 0$.

3. Evaluate given definite integral as limit of sum :

$$\int_0^4 (x + e^{2x})dx$$

Or

Integrate the rational function :

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$$

4. Find the shortest distance between the lines :

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Or

Find the distance of a point (2, 5, –3) from the plane :

$$\bar{r}.\left(6\hat{i} - 3\hat{j} + 2\hat{k}\right) = 4$$

5. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X ? Is X a random variable ?

Or

Bag I contains 3 red and 4 black balls and Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bag and is found to be red. Find the probability that it was drawn from Bag II.

(Short Answer Type Questions) 4 each

6. If :

$$f(x) = \frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3},$$

show that $f \circ f(x) = x$, for all $x \neq \frac{2}{3}$. What is the inverse of f ?

7. Write the function in the simplest form :

$$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), a > 0; \frac{-a}{\sqrt{3}} \leq x \leq \frac{a}{\sqrt{3}}$$

8. Prove that any square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrix.

9. Find $\frac{dy}{dx}$ in $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$.

10. Find the equation of all lines having slope 2 which are tangents to the curve $y = \frac{1}{x-3}, x \neq 3$.

11. Find both the maximum value and minimum value of $3x^4 - 8x^3 + 12x^2 - 48x - 25$ on the interval $[0, 3]$.

12. Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$; $y = 3x + 1$; $x = 4$.

13. Show that the differential equation is homogeneous and solve it :

$$\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$$

14. Find the co-ordinates of the foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z - 6 = 0$.

15. Solve the Linear Programming problem graphically :

Minimize $z = 3x + 5y$

such that :

$$x + 3y \geq 3$$

$$x + y \geq 2$$

$$x, y \geq 0$$

(Very Short Answer Type Questions) 2 each

16. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27\ |A|$.

17. Examine the function for continuity $f(x) = x - 5$.

18. Evaluate :

$$\int \frac{dx}{\sqrt{2x - x^2}}$$

19. Evaluate :

$$\int_{-1}^{1} (x+1)\,dx$$

20. Verify that the given function is a solution of the corresponding differential equation :

$$y = x^2 + 2x + c$$

$$y' - 2x - 2 = 0$$

21. Find the the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

22. Write two different vectors having same direction.

23. Find the unit vector in the direction of a vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$.

24. Minimize $Z = 3x + 2y$ subject to the constraints $x + y \geq 8$, $x, y \geq 0$.

25. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$.

(Objective Type Questions) 1 each

26. (i) Define transitive relation.

(ii) Find the value of :

$$\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$$

(iii) If A and B are two skew-symmetric matrices of same order, then AB is symmetric matrix of .... (Fill in the blank)

(iv) The derivative of $\sin x$ w.r.t. $\cos x$ is ....

(Fill in the blank)

(v) $\int \frac{\sin^6 x}{\cos^8 x} dx = \frac{\tan^7 x}{7} + c$ (True/False)

(vi) $\int_{-a}^{a} f(x) dx = 0$, if f is an odd function. (True/False)

(vii) The order of the differential equation of all circles of given radius a is :

(a) 1 (b) 2

(c) 3 (d) 4

(viii) The value of λ for which the two vectors $2\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\lambda\hat{j}+\hat{k}$ are perpendicular is :

(a) 2 (b) 4

(c) 6 (d) 8

(ix) If $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then $P(B/A) + P(A/B)$ equals :

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{5}{12}$ (d) $\frac{7}{2}$

(x) For the following probability distribution :

X	–4	–3	–2	–1	0
P(X)	0.1	0.2	0.3	0.2	0.2

E(X) is equal to :

(a) 0 (b) –1

(c) –2 (d) –1.8