

**B-21-A**

Roll No.....

Total No. of Questions : 26]

[Total No. of Printed Pages : 8 + Graph

**HSEIIRWZJO17****15221-A****MATHEMATICS**

Time : 3 Hours]

[Maximum Marks : 100

(Long Answer Type Questions)

6 each

1. If  $x, y, z$  are different and

$$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0,$$

then show that  $1 + xyz = 0$ .*Or*

If :

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

verify that  $A^3 - 6A^2 + 9A - 4I = 0$  and hence find  $A^{-1}$ .

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Turn Over

( 2 )

2. Find  $\frac{dy}{dx}$ , if  $y^x + x^y + x^x = a^b$ .

Or

If  $x = \sqrt{a^{\cos^{-1}t}}$   $y = \sqrt{a^{\sin^{-1}t}}$ , show that :

$$\frac{dy}{dx} = -\frac{y}{x}$$

3. Evaluate :

$$\int_0^{\pi/2} \sqrt{\sin \phi} \cos^5 \phi d\phi$$

Or

Evaluate :

$$\int \frac{(3 \sin \phi - 2) \cos \phi}{5 - \cos^2 \phi - 4 \sin \phi} d\phi$$

4. Find the value of P so that the lines :

$$\frac{1-x}{3} = \frac{7y-14}{2P} = \frac{z-3}{2}$$

and

$$\frac{7x-7}{3P} = \frac{y-5}{1} = \frac{6-z}{5}$$

are at right angles.

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Find the vector equation of the plane passing through the intersection of the planes  $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$  and  $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$  and the point (1, 1, 1).

5. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective ?

Or

Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of the number of kings.

(Short Answer Type Questions)

4 each

6. Write  $\tan^{-1} \left( \frac{\cos x - \sin x}{\cos x + \sin x} \right)$ ,  $0 < x < \pi$  in the simplest form.

7. Consider  $f : \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = 4x + 3$ . Show that  $f$  is invertible. Find the inverse of  $f$ .

8. Find  $x$  and  $y$ , if :

$$x + y = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix}$$

and

$$x - y = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix}$$

9. Show that :

$$y = \log(1+x) - \frac{2x}{2+x}, \quad x > -1,$$

is an increasing function of  $x$  throughout its domain.

10. Find the relation between  $a$  and  $b$  so that the function  $f$  is defined by

$$f(x) = \begin{cases} ax+1, & \text{if } x \leq 3 \\ bx+3, & \text{if } x > 3 \end{cases}$$

is continuous at  $x = 3$ .

11. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference ?

12. Find a unit vector perpendicular to each of the vector  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$ , where  $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ .

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13. Find the general solution of the differential equation :

$$x \frac{dy}{dx} + 2y = x^2 (x \neq 0)$$

14. Find the area of the region bounded by  $y^2 = 9x$ ,  $x = 2$ ,  $x = 4$  and the  $x$ -axis in the first quadrant.

15. Determine graphically the minimum value of the objective function

$$Z = -50x + 20y$$

Subject to the constraints :

$$2x - y \geq -5;$$

$$3x + y \geq 3;$$

$$2x - 3y \leq 12,$$

$$x \geq 0, y \geq 0$$

(Very Short Answer Type Questions)

16. Compute :

$$\begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

17. Find  $\frac{dy}{dx}$ , if  $2x + 3y = \sin x$ .

18. Use differential to approximate  $(25)^{1/3}$ .

19. Find the slope of tangent to the curve  $y = 3x^2 - 4x$  at  $x = 4$ .

20. Find  $\frac{dy}{dx}$ , if  $x = at^2$ ,  $y = 2at$ .

21. Find the general solution of the differential equation :

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

22. Find the direction cosines of the vector  $\hat{i} + 2\hat{j} + 3\hat{k}$ .

23. Find the projection of vector  $\hat{i} - \hat{j}$  on the vector  $\hat{i} + \hat{j}$ .

24. Find  $|\vec{a} \times \vec{b}|$ , if

$$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$$

and

$$\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

25. Compute  $P\left(\frac{E}{F}\right)$ , if  $P(F) = 0.3$  and  $P(E \cap F) = 0.2$ .

(Objective Type Questions)

1 each

26. (i) Define diagonal matrix.

(ii) Define the term feasible region.

(iii) What do you mean by manufacturing problems ?

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- (iv) Give an example of a relation which is symmetric but neither reflexive nor transitive.
- (v) The direction ratios of the line joining the points (4, 3, -5) and (-2, 1, -8) are .....
- (vi) The probability of obtaining an even prime number on each die, when a pair of dice is rolled is .....
- (vii) A coin is tossed twice, then the probability that at least one tail occurs is  $\frac{3}{4}$ . <https://www.jkboseonline.com>

(viii)  $\frac{d}{dx} [\log \cdot \sin x] = \tan x.$

(True/False)

(ix)  $\sin \left( \frac{\pi}{3} - \sin^{-1} \frac{1}{2} \right)$  is equal to :

(a)  $\frac{1}{2}$

(b)  $\frac{1}{3}$

(c)  $\frac{1}{4}$

(d) 1

(Choose the correct option)

(x)  $\int \sec x \, dx$  is equal to :

(a)  $\log | \sec x + \tan x | + C$

(b)  $\log | \sec x - \tan x | + C$

(c)  $\log | \sec x + \tan x |$

(d) None of these

(Choose the correct option)