B–7–Y

Total No. of Questions : 30] [Total No. of Printed Pages : 8

12th ARM(SZ)JKUT2024
1107–Y
MATHEMATICS

Time : 3 Hours] [Maximum Marks : 80

SECTION–A
(OBJECTIVE TYPE QUESTIONS/ MULTIPLE CHOICE QUESTIONS)

1 each

1. In determinant

$$\begin{vmatrix} -1 & 3 \\ 6 & 10 \end{vmatrix}$$

cofactor of 10 is

(A) 1

(B) –1

(C) 3

(D) 6

2. If matrix B is inverse of matrix A, then AB = BA = ...

Turn Over

3. Order of differential equation

$$x^2 \frac{dy}{dx} = -y^5$$

is

(A) 1

(B) 2

(C) 5

(D) 0

4. Derivative of $\cos(xe^x)$ is :

(A) $-e^x (x + 1) \sin(xe^x)$

(B) $e^x (x + 1) \sin(xe^x)$

(C) $-\sin(xe^x)$

(D) $\sin(xe^x)$

5. Number of points of discontinuity of the function $f(x) = x + 5$ is :

(A) 1

(B) 2

(C) 3

(D) 0

6. $\int \tan x \, dx$ is equal to $\log \sec x + c$. (True/False

7. The function $f(x) = x^2 + 2x + 3$ is decreasing for ..... .

8. Direction cosines of y-axis are :

(A) 1, 0, 0

(B) 0, 1, 0

(C) 0, 0, 1

(D) 0, 0, 0

9. Unit vector along the direction of the vector $\hat{i} + \hat{j} + \hat{k}$ is :

(A) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

(B) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{2}}$

(C) $\frac{\hat{i} + \hat{j} + \hat{k}}{3}$

(D) $\frac{\hat{i} + \hat{j} + \hat{k}}{2}$

10. Define feasible region.

SECTION–B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

11. Check the injectivity and subjectivity of the function $f : R \to R$ given by $f(x) = x^2$.

12. Find the principal value of

$$\tan^{-1}(-\sqrt{3}).$$

13. Prove that the function $f(x) = \log \cos x$ is decreasing for

$$\left(0, \frac{\pi}{2}\right).$$

14. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$.

15. Show that :

$$(\vec{a} - \vec{b}) \times (a + \vec{b}) = 2(\vec{a} \times \vec{b}).$$

16. Find :

$$\int \frac{1}{x + x \log x} dx.$$

17. Evaluate :

$$\int_0^{\pi/2} \cos 2x \, dx.$$

8. If

$$P(A) = \frac{3}{5}$$

and

$$P(B) = \frac{1}{5}$$

find $P(A \cap B)$ if A and B are independent events

9. Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$

10. Show that the matrix

$$A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

is a symmetric matrix.

SECTION-C

(SHORT ANSWER TYPE QUESTIONS) 4 each

1. Find the general solution of the differential equation

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x.$$

2. Evaluate :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx.$$

23. If

$$x = \sqrt{a^{\sin^{-1} t}}, \; y = \sqrt{a^{\cos^{-1} t}}$$

show that :

$$\frac{dy}{dx} = -\frac{y}{x}.$$

24. Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda (\hat{i} - \hat{j} - \hat{k})$$

and

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu (2\hat{i} + \hat{j} + 2\hat{k}).$$

25. If

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \; \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

and

$$\vec{c} = 3\hat{i} + \hat{j}$$

are such that :

$$\vec{a} + \lambda \vec{b}$$

is perpendicular to $\vec{c}$, then find the value of λ.

26. Solve the linear programming problem graphically :

Minimize $z = -3x + 4y$

subject to :

$$x + 2y \leq 8,$$

$$3x + 2y \leq 12,$$

$$x \geq 0,\ y \geq 0.$$

27. Show that the relation R in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$ is an equivalence relation.

28. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

SECTION–D

(LONG ANSWER TYPE QUESTIONS) 6 each

29. If :

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

verify A (adj A) = (adj A)A = |A| I.

Or

Let

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

verify $(AB)^{-1} = B^{-1}.A^{-1}$.

30. Evaluate :

$$\int_{-1}^{1} 5x^4 \sqrt{x^5 + 1}\, dx.$$

Or

Evaluate :

$$\int \frac{(3x - 1)\, dx}{(x - 1)(x - 2)(x - 3)}.$$

31. If $y = Ae^{mx} + Be^{nx}$, show that :

$$\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = 0.$$

Or

Differentiate w.r.t. x :

$$(\log x)^x + x^{\log x}.$$