**B–5–B**

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# XIISZRJDF20

# 1105–B

# MATHEMATICS

Time : 3 Hours] [Maximum Marks : 100

**(Long Answer Type Questions)** 6 each

1. Using properties of determinants, show that :

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$$

*Or*

Solve the following system of equations by matrix method :

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

Turn Over

2. Differentiate the function :

$$x^{\sin x} + (\sin x)^{\cos x}$$

*Or*

If $y = 3\cos(\log x) + 4\sin(\log x)$, show that :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

3. Evaluate :

$$\int \frac{\cos x \, dx}{(1-\sin x)(2-\sin x)}$$

*Or*

Find $\int_0^2 (x^2+1)\,dx$ as the limit of a sum.

4. Show that the differential equation $(x^2 + xy)\,dy = (x^2 + y^2)\,dx$ is homogeneous and solve it.

*Or*

Find the general solution of the differential equation :

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

5. Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

and $$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

*Or*

Find the angle between the planes whose vector equations are :

$$\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$$

and $$\vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$$

**(Short Answer Type Questions)** 4 each

6. Find the value of K so that the function "$f$" defined by :

$$f(x) = \begin{cases} Kx + 1, & \text{if } x \leq 5 \\ 3x - 5, & \text{if } x > 5 \end{cases}$$

is continuous at $x = 5$.

7. If $f(x) = \dfrac{4x+3}{6x-4}$, $x \neq \dfrac{2}{3}$, show that $f \circ f(x) = x$, for all $x \neq \dfrac{2}{3}$. What is the inverse of '$f$' ?

8. If $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$, then find the value of $x$.

9. Using the properties of determinants and without expanding, prove that :

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}=0$$

10. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

11. If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by :

$$1 - P(A')\ P(B')$$

Prove it.

12. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i}+5\hat{j}-6\hat{k}$.

13. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

14. Evaluate :

$$\int_0^1 \frac{x\,dx}{x^2+1}$$

15. Solve the following linear programming problems graphically :

Maximise : $z = 5x + 3y$

Subject to

$$3x + 5y \leq 15$$

$$5x + 2y \leq 10$$

$$x \geq 0,\ y \geq 0$$

**(Very Short Answer Type Questions)** 2 each

16. Express $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \neq 0$ in the simplest form.

17. Find the value of $x$ and $y$, if :

$$x\begin{pmatrix}2\\3\end{pmatrix} + y\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}10\\5\end{pmatrix}$$

18. Find $\frac{dy}{dx}$, if $2x + 3y = \sin y$.

19. Find $\frac{dy}{dx}$, if $x = a \sec \theta$, $y = b \tan \theta$.

20. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

21. Find the angle between the vectors $\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$.

22. Shade the feasible region of L.P.P. :

    $x + 2y \leq 10$, $3x + y \leq 15$, $x, y \geq 0$.

23. If P(A) = 0.8, P(B) = 0.5 and P(B/A) = 0′ find P(A/B).

24. Evaluate :

$$\int x \sin 3x \, dx$$

25. Evaluate :

$$\int_2^3 \frac{dx}{x^2 - 1}$$

**(Objective Type Questions)** **1 each**

26. (i) Define skew-symmetric matrix.

    (ii) Area lying between the curves $y^2 = 4x$ and $y = 2x$ is $\frac{1}{3}$.

(True/False)

(iii) $\int(\text{cosec}^2 x - \cot^2 x) =$ .................. (Fill in the blank)

(iv) The degree of the differential equation

$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) = 0$ is 2. (True/False)

(v) $\int\sqrt{a^2 - x^2}$ = ............. . (Fill in the blank)

(vi) Direction cosines of $y$-axis are ............ . (Fill in the blank)

(vii) The rate of change of the area of a circle with respect to its radius '$r$', when $r = 3$ cm is : 

(A) 4 cm²/cm (B) 6 cm²/cm

(C) 8 cm²/cm (D) None of these

(viii) The slope of tangent to the curve $y = 3x^4 - 4x$ at $x = 4$ is :

(A) 760 (B) 762

(C) 764 (D) None of these

(ix) $\int e^x \sec x (1 + \tan x)\, dx$ is equal to :

(A) $e^x \cos x + C$ (B) $e^x \sec x + C$

(C) $e^x \sin x + C$ (D) $e^x \tan x + C$

(x) The integrating factor of the differential equation $y\, dy - (x + 2y^2)\, dx = 0$ is :

(A) $\frac{1}{y}$ (B) $\frac{1}{y^2}$

(C) $\frac{1}{x}$ (D) None of these